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# Application of Frequency B-Spline Wavelets for Detection of Defects in Rolling Bearings

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## Abstract

Frequency B-spline (fbsp) wavelets have been applied in the present work for detection of localized defects in the inner race of a rolling element bearing. The theoretical response from the bearing system due to an inner race defect coupled with additive noise of varying magnitude form the simulated signal. Methodology of defect detection includes three steps. Firstly, wavelet transformation of noisy vibration signal has been obtained using fbsp wavelets with appropriate parameters. Then, maximum wavelet coefficients were retained from the result of transformation on which a threshold level was applied. Finally fast Fourier transformation was applied on these retained coefficients to identify the presence of components at characteristic frequency corresponding to the defect location. The results show that the method could specifically identify bearing defects even from noisy vibration signals.

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**Keywords:** Rolling bearing; vibration; defect; frequency B-spline wavelet

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## 1. Introduction

Vibration signal analysis is probably the most widely used technique for condition monitoring of rolling element bearings. The raw vibration signal can be processed through the inbuilt functions of the instruments or routed through advanced signal processing techniques to extract the required information. A review of different vibration and acoustic measurement techniques for condition monitoring of rolling bearings has been presented in Ref. [1].

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Bearing fault diagnosis can be carried out in time domain, frequency domain and time-frequency domain. Time domain method mostly involves the application of statistical methods, kurtosis being the most effective among them [2]. However, the advent of fast Fourier transform (FFT) analyzers resulted in wider use of the frequency domain approach. Interaction of bearing defect with mating element results in generation of repetitive pulses at a frequency termed as characteristic defect frequency which depend on the bearing kinematics [1]. The expressions for characteristic defect frequencies have been presented in Table 1. The objective of frequency domain approach was to identify spectral components at characteristic defect frequency to confirm the presence of defect at the corresponding element. Some additional signal processing techniques such as envelope detection or high frequency resonance technique [3] has been often employed to improve the signal to noise ratio. Spectral analysis through Fourier transform has been found to be very effective for stationary processes. However, the effectiveness of Fourier transform as a tool gets significantly reduced when the periodicity is disturbed [4]. The noise emanated from adjacent machinery and supports, also masks the signal and adversely affects the analysis. This fact has led to the development of time-frequency analysis methods such as wavelet analysis.

Wavelet analysis is the presentation of signals in time-frequency distribution diagrams with multi-resolution in time and frequency and is thus inherently better-suited for capturing transient events in an otherwise periodic signal, as are commonly seen in a defective bearing. A number of researchers [5, 6, 7, 8, 9] have investigated the ability of wavelet transform in extracting defect features from bearing vibration signals using different wavelets available in the literature. Attempt has also been made to customize the mother wavelet from system's perspective for detection of bearing defect [10]. Application of Morlet wavelet has been mostly favored by researchers [7, 8] for analyzing defect-induced vibration signals from bearings. Compared to Morlet wavelets, frequency B-spline wavelets have one additional parameter to define their shape. However, the potential of spline wavelet as a bearing-diagnostic tool has not yet been significantly explored. In the present work, therefore, frequency B-spline (fbsp) wavelets have been applied to simulated vibration signal from a bearing with localized defect on the inner race. The defect detection methodology also involves coupling of wavelet transformation (WT) of vibration signal and fast Fourier transformation (FFT) of wavelet coefficients to rip in the benefits of both the techniques.

**Table 1:** Characteristic defect frequencies for a rolling element bearing ( $f_s$ : shaft frequency;  $d$ : rolling element diameter;  $D$ : Pitch diameter;  $\beta$ : Nominal contact angle;  $Z$ : Number of rolling elements)

Characteristic frequency	Expression
Cage frequency, $f_c$	$(f_s/2)[1 - (d/D) \cos \beta]$
Outer race defect frequency, $f_{od}$	$(Z f_s/2)[1 - (d/D) \cos \beta]$
Inner race defect frequency, $f_{id}$	$(Z f_s/2)[1 + (d/D) \cos \beta]$
Rolling element defect frequency, $f_{red}$	$(D f_s/d)[1 - (d/D)^2 \cos \beta]$

## 2. Procedure of signal analysis

In this section, the features and specifications of fbsp wavelets have been described. Generation of simulated signal to represent defect-induced vibration signals has also been discussed. The methodology of coupling WT and FFT for extraction of diagnostic features has also been included in this section.

### 2.1. Frequency B Spline wavelet

The frequency B spline wavelets can be defined directly in the frequency domain in terms of integer order, central frequency and bandwidth. The expression for Complex Frequency B-Spline wavelet function is given as:

$$\psi(t) = \sqrt{F_b} \left( \text{sinc} \left( \frac{F_b t}{m} \right) \right)^m e^{2i\pi F_c t} \quad (1)$$

$$\text{sinc}(*) = \begin{cases} 1 & *=0 \\ \frac{\sin *}{*} & \text{otherwise} \end{cases}$$

In the above expression,  $m$ ,  $F_b$  and  $F_c$  are integer order parameter, bandwidth parameter and wavelet central frequency respectively.

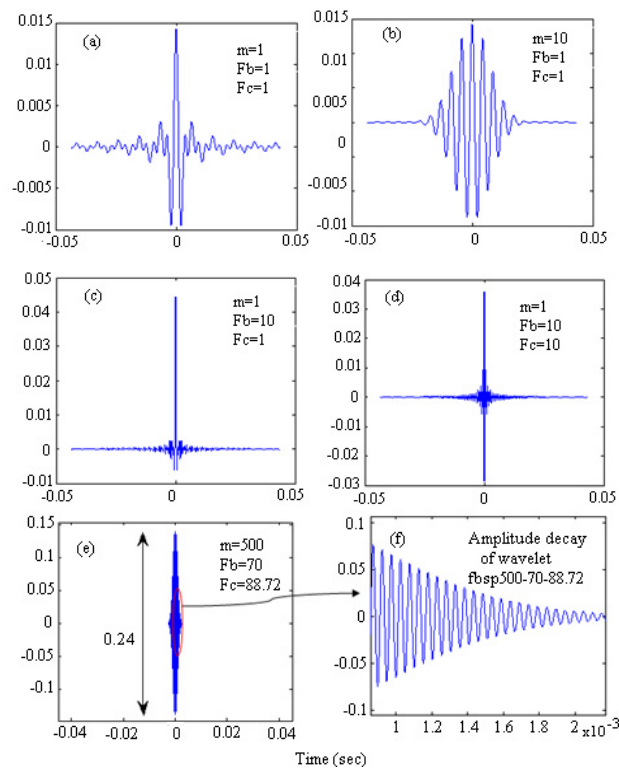


Fig.1. Shape of spline wavelet (a) fbsp(1-1-1); (b) fbsp(10-1-1); (c) fbsp(1-10-1); (d) fbsp(1-10-10);  
(e) fbsp(500-70-88.72); (f) enlarged view of fbsp(500-70-88.72)

Spline wavelets for different values of  $m$ ,  $F_b$  and  $F_c$  are shown in Fig. 1. It is evident from the figures that the wavelet resembles a transient signal and hence can be suitably applied for analyzing vibration signals from defective bearing. Appropriate choice of parameters, viz., integer order, bandwidth and central frequency, for the wavelet can lead to high magnitudes of coefficients resulting in better interpretation of results.

## 2.2. Generation of Simulated Signals

For generation of simulated signals, the bearing system has been modeled as a discrete spring-mass-dashpot system as proposed by Sassi et al. [11]. The simplified model is shown in Fig. 2 and considers only on the flexural vibration of races in the principal direction, i.e., the direction of maximum deflection of bearing.

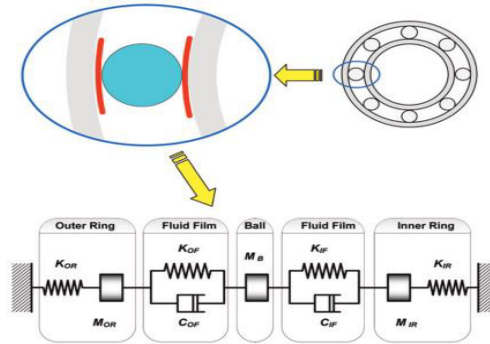


Fig.2 Vibratory model of bearing system

In this model,  $m_{OR}$ ,  $m_{IR}$  and  $m_R$  denote masses of outer race, inner race and rolling element respectively.  $K_{OR}$  and  $K_{IR}$  denote stiffness for outer and inner races respectively.  $K_{OF}$  and  $C_{OF}$  represent the stiffness and damping coefficients of the fluid film between the outer race and rolling element.  $K_{IF}$  and  $C_{IF}$  represent the same for the fluid film between the inner race and the rolling element. All the values of masses, stiffness and damping coefficients can be determined following the procedure laid down in Ref. [11]. The equation of motion for the above system due to an inner race defect can be expressed as:

$$\begin{bmatrix} m_{OR} & 0 & 0 \\ 0 & m_R & 0 \\ 0 & 0 & m_{IR} \end{bmatrix} \begin{Bmatrix} \ddot{y}_{OR} \\ \ddot{y}_R \\ \ddot{y}_{IR} \end{Bmatrix} + \begin{bmatrix} c_{OF} & -c_{OF} & 0 \\ -c_{OF} & c_{OF} + c_{IF} & -c_{IF} \\ 0 & -c_{IF} & c_{IF} \end{bmatrix} \begin{Bmatrix} \dot{y}_{OR} \\ \dot{y}_R \\ \dot{y}_{IR} \end{Bmatrix} + \begin{bmatrix} k_{OR} + k_{OF} & -k_{OF} & 0 \\ -k_{OF} & k_{OF} + k_{IF} & -k_{IF} \\ 0 & -k_{IF} & k_{IF} + k_{IR} \end{bmatrix} \begin{Bmatrix} y_{OR} \\ y_R \\ y_{IR} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{IR} \end{Bmatrix} \quad (2)$$

In the above equation,  $y$  and  $Q$  denote displacement and excitation of the appropriate element. Solution for the above equation has been obtained using transfer function approach. Considering Laplace transform of Eq. (2), the solution has been obtained using MATLAB Simulink block diagram as shown in Fig. 3. Response of the outer race due to an inner race defect has been considered in the block diagram.

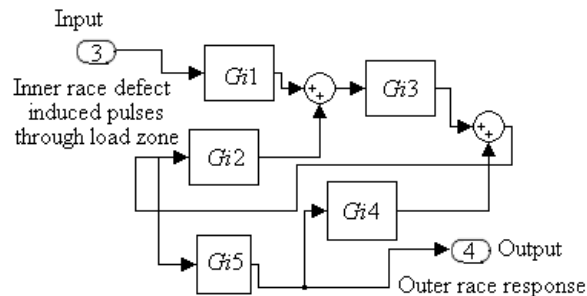


Fig.3. Simulink block diagram of transfer functions associated with an inner race defect

The transfer functions of individual blocks depicting the roles of bearing elements and their vibratory properties have been enlisted in Table 2.

Table 2: Transfer functions associated to Rolling Element Defect

$Gi1$	$\frac{1}{m_{IR} s^2 + c_{IF} s + (k_{IR} + k_{IF})}$
$Gi2$	$\frac{(c_{IF} s + k_{IF})}{m_{IR} s^2 + c_{IF} s + (k_{IR} + k_{IF})}$
$Gi3$	$\frac{(c_{IF} s + k_{IF})}{m_R s^2 + (c_{IF} + c_{OF}) s + (k_{IF} + k_{OF})}$
$Gi4$	$\frac{(c_{OF} s + k_{OF})}{m_R s^2 + (c_{IF} + c_{OF}) s + (k_{IF} + k_{OF})}$
$Gi5$	$\frac{(c_{OF} s + k_{OF})}{m_{OR} s^2 + c_{OF} s + (k_{OR} + k_{OF})}$

In the present work, simulated signal has been generated based on the following equation [3, 12]:

$$y = (y_f \cdot y_q) * y_{bs} + n(t) \quad (3)$$

In the above equation, impulses generated due to interaction of inner race defect with mating elements,  $y_f$ , get modulated by the radial load,  $y_q$  and the product is convolved with the response function,  $y_{bs}$  involving vibratory elements of bearing. Addition of noise,  $n(t)$ , due to machine and structure born vibration as well as noise from other sources to the response obtained from bearing results in generating the simulated signal. The process of simulation in MATLAB Simulink is depicted in the block diagram shown in Fig. 4.

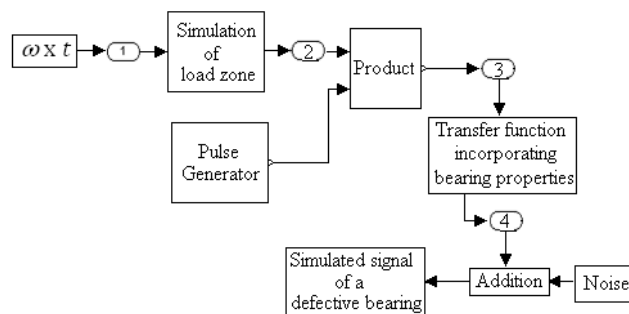


Fig. 4. Simulink block diagram for generation of simulated signal

### 2.3 Methodology for Defect Detection

Diagnostic features have been extracted using a coupled method of continuous wavelet transform (CWT) and fast Fourier transform (FFT). The complete process is depicted in the flowchart shown in Fig. 5. The process starts with optimization of fbsp wavelets [13, 14]. CWT of the simulated signal is then carried out using the optimized wavelets. Wavelet coefficients are thus generated, the magnitudes of which will depend on the selection and optimization of wavelet. In the next step, only the maximum wavelet coefficients are retained. Subsequently a threshold is decided and the coefficients exceeding the threshold are carried forward for the next operation of FFT. Thus FFT is applied on a much reduced number of wavelet coefficients resulting in reduced computation time as well as reduced effect of noise level. The resulting spectrum is investigated for the presence of significant component at inner race defect frequency,  $f_{id}$ .

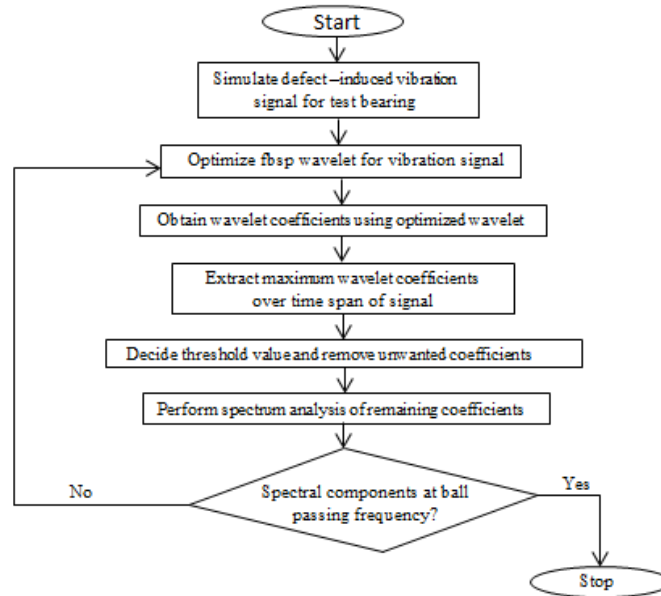


Fig. 5. The methodology for defect detection

## 3. Result and Discussion

In order to obtain numerical results, an NJ 305 cylindrical roller bearing has been considered. Dimensions of NJ 305 bearing are as follows: 25 mm bore, 62 mm outside diameter, 17 mm width and 43.5 mm pitch diameter. They have 10 rollers each of 8.5 mm diameter. A shaft speed of 2710 rpm has been considered for this study. For the bearing under consideration, the shaft frequency at this speed is 45.16 Hz and inner race defect frequency, is equal to 270 Hz.

### 3.1 Simulated Signal

Simulated signal for the bearing with inner race defect has been obtained following the procedure laid down in section 2.2. Initially the signal has been obtained without noise and then the additive noise level was gradually increased till the level reached  $-9.73$  dB. The simulated signal at 2710 rpm with noise level of  $-9.73$  dB is shown in Fig. 6.

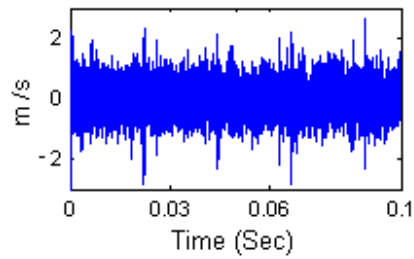


Fig.6. Simulated signal for NJ 305 bearing at 2710 rpm with noise level of  $-9.73$  dB

### 3.2. Optimization of wavelet and wavelet transformation

Frequency B-spline wavelets require three parameters, viz., integer order  $m$ , bandwidth  $fb$  and central frequency  $fc$  for its specification. In this study, these parameters have been optimized based on mutual information and determinant of correlation coefficient matrix [13, 14]. The optimized wavelet has been identified as  $500 - 70 - 88.70$ , i.e. the wavelet has an order of 500, bandwidth of 70 Hz and central frequency of 88.70 Hz. CWT of the simulated signal has been obtained using the optimized wavelet and a temporal representation of wavelet coefficients is shown in Fig. 7.

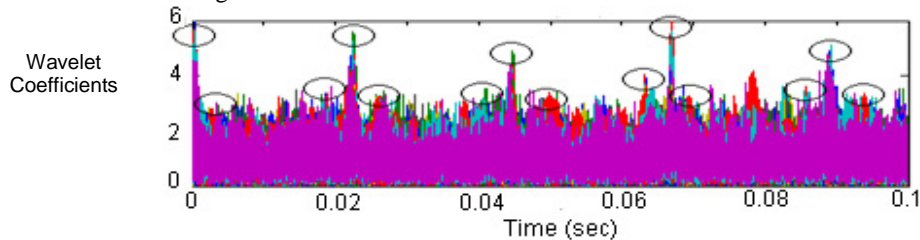


Fig. 7. A plot of wavelet coefficients vs. time for simulated signal of NJ 305 bearing

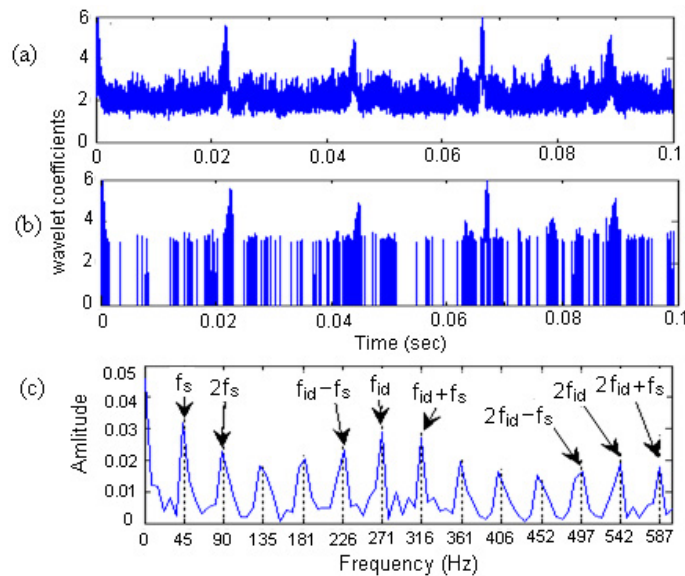


Fig. 8. (a) Maximum wavelet coefficients vs time (b) Coefficients above threshold value vs time (c) Frequency spectrum of wavelet coefficients above threshold value

### 3.3. Extraction of Diagnostic Features

Defect detection methodology followed the flowchart shown in Fig. 5. Only the maximum wavelet coefficients at each step time have been retained and other coefficients have been eliminated. A plot of the maximum coefficients vs. time is shown in Fig. 8(a). A threshold level of 3 has been considered and the coefficients which have a value less than 3 have been eliminated. Thus fewer coefficients remain for further processing and these are shown in Fig. 8(b). FFT of these remaining wavelet coefficients shown in Fig. 8(b) resulted in the spectrum shown in Fig. 8(c). Fig. 8(c) shows significant peaks at inner race defect frequency  $f_{id}$  and its harmonics with sidebands at multiples of shaft frequency,  $f_s$ . Components at shaft frequency as well as the sidebands about  $f_{id}$  and its harmonics are due to the modulation of impulses by load at the defect position, which is periodic in nature with fundamental frequency at  $f_s$ . The spectrum is very similar in nature to those obtained for inner race defect through analysis in pure frequency domain [3], [15].

### 4. Conclusion

In the present work, effectiveness of frequency B-spline wavelet in detection of defect in the inner race of a rolling element bearing has been investigated. The method of detection involves coupling of continuous wavelet transformation using fbsp wavelets followed by fast Fourier transformation of processed wavelet coefficients. The results for a simulated noisy vibration signal show that the spectrum has significant components at inner race defect frequency and its harmonics with sidebands at multiples of shaft frequency. Presence of components at harmonics of inner race defect frequency indicates that the bearing has defect on the inner race. Components and sidebands at multiples of shaft frequency have resulted due to modulation of defect-induced impulses by periodic load zone. The results show that the method is effective in detection of inner race defect from vibration signals with very high level of noise.

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